The Uncertainty-Investment Relationship with Endogenous Investment Size

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Abstract

This paper revisits the uncertainty-investment relationship with a real-option model where the firm chooses both the timing and the size of the investment, unlike existing studies. Using a new measure of investment that takes into account both size and timing, it is shown that, in general, investment is a non-monotonic (initially increasing and subsequently decreasing) function of uncertainty. Thus, uncertainty can have a positive or negative impact on investment; however, it is more likely to be positive when demand growth rate and demand volatility are low, and discount rate and operating cost are high. The relationship also depends on the production technology; with decreasing-returns-to-scale technology it is more likely to be positive, and with increasing-returns-to-scale technology it is more likely to be negative.

Keywords: Uncertainty; Volatility; Investment; Real-option model.

JEL classification: D2, G3.

Introduction

Capital investment is a critical decision for both the corporation (because it is the most important means to create value, and incorrect or sub-optimal investment decisions can have serious negative consequences for shareholder wealth) as well as the broader economy (because corporate investment is a fundamental determinant of economic growth, economic progress, and standard of living).

One of the most significant factors in the corporate investment decision is *uncertainty* or *volatility* (Balcer and Lippman, 1984, Farzin et al., 1998, McCardle, 1985, Murto, 2007, etc). Not surprisingly, there is a substantial literature, both theoretical and empirical, on the effect of uncertainty on investment.

The empirical literature on the uncertainty-investment relationship is mixed, as shown by the brief (and not at all exhaustive) summary of the literature below. While a number of papers report a negative relationship between uncertainty and investment, some find a positive, ambiguous or non-monotonic relationship. Kellogg (2014) finds that an increase in oil price volatility causes firms to reduce drilling activity, Stein and Stone (2013) and Pennings and Altomonte (2006) find that higher levels of uncertainty result in reduced capital investment, while Serven (2003) finds a negative relationship between exchange-rate volatility and investment in most of their samples. On the other hand, Bloom et al. (2006) find that the effect of uncertainty (measured by demand shock) on investment is weaker for firms with high levels of uncertainty, and that the response of investment to demand shocks is convex; Bloom (2000) finds that the real-option effects of uncertainty play no role in the long-run rate of investment; and Misund and Mohn (2006) find that uncertainty has a significant effect on investment in the oil and gas industry, but the sign of the effect is not settled. Jeanneret (2007) finds a U-shaped relationship between investment and exchange-rate volatility, that is, investment is initially decreasing and subsequently increasing in uncertainty. Driver et al. (2008) find that uncertainty might have a positive or negative effect on investment, depending on the industry.

The theoretical literature on the uncertainty-investment relationship is also not quite unanimous. As stated by Leahy and Whited (1996): " . . . taken as a whole, what theory has to say is ambiguous. Different

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theories emphasize different channels, some pointing to a positive relationship and some to a negative relationship" (p. 64). The earlier strand of the literature, based on adjustment costs, e.g., Hartman (1972) and Abel (1983), predicts a positive effect of uncertainty on investment. The more recent "real options" literature (McDonald and Siegel, 1986, Dixit, 1989, Dixit and Pindyck, 1994, etc), which views investment as the exercise of a call option, is by now well-established as a legitimate approach to investment analysis (Driouchi et al., 2009). When an investment opportunity is viewed as a real option, the effect of uncertainty on investment timing is unambiguous – higher volatility results in delayed option exercise or delayed investment, which is a standard result in option theory; thus, uncertainty has a negative effect on investment. In fact, Dixit and Pindyck (1994) state: "Hence, greater uncertainty reduces investment" (p. 192). However, Sarkar (2000) and Wong (2007) demonstrate that the relationship is generally non-monotonic, and Alvarez and Koskela (2006) show that interest-rate volatility can have a positive or a negative effect on investment, depending on the level of interest rate. Thus, both the theoretical predictions and empirical evidence on this issue are not settled in the literature.

This paper takes a new look at the uncertainty-investment relationship in a real-option setting, but with one significant departure from the existing literature – the investment size or scale is not fixed but chosen optimally by the firm. Endogeneity of investment size could potentially impact the uncertainty-investment relationship because of two reasons: (i) uncertainty itself might have a direct effect on optimal size because of the asymmetric nature of volatility under limited liability (see, for instance, Sarkar, 2009); and (ii) greater uncertainty will undoubtedly result in delayed investment; however, delayed investment will in turn lead to larger investment size, as shown by Bar-Ilan and Strange (1999), Huberts et al. (2015), and Hagspiel et al. (2016). Thus, a higher volatility will indirectly result in a larger investment, which implies a positive uncertainty-investment relationship.

With fixed-size investment, greater uncertainty will have an unambiguously negative impact on investment because of the timing effect. However, allowing the firm to choose investment size will complicate matters because of the size effect mentioned in the previous paragraph, and the overall effect of uncertainty on investment will no longer be unambiguous, because of the conflicting "timing" and "size" effects. If investment is delayed but larger, the overall effect is ambiguous because delayed investment represents a negative effect while larger investment represents a positive effect, on investment. The offsetting effects of timing and size make it difficult to determine the overall effect of uncertainty on investment; in order to overcome this difficulty, we have to look for a composite measure that accounts for both timing and size effects, and therefore allows us to make a fair comparison of investment as volatility is varied. For this purpose, we choose the measure EPVI (the expected present value of investment), described in Section 2.6.

The main result from our model is that investment (measured by EPVI) is, in general, initially an increasing and subsequently a decreasing function of uncertainty. However, the exact uncertainty level at which it switches from increasing to decreasing varies widely with parameter values. Uncertainty is more likely to have a positive effect on investment when demand growth rate and demand volatility are low, and when the decreasing-returns-to-scale parameter, discount rate and operating cost are high.

The rest of the paper is organized as follows. Section 2 develops the model, identifies the optimal investment policy (trigger and size) and develops a measure of investment magnitude that takes into account both timing and size effects. Section 3 presents the results of the model, and Section 4 summarizes the results and concludes.

The Model

We use a simple real-option model of investment to study the uncertainty-investment relationship, similar to traditional models such as Dixit and Pindyck (1994). The distinguishing feature of our model is that investment size is endogenous,

[1] There are also some papers that take a game-theoretic or strategic approach to real options, where one firm's investment decision will affect other firms' payoffs, e.g., Nielsen (2002), or Martzoukos and Zacharias (2013). However, our model has no strategic implications because the firm's payoffs are determined by its own decisions and its own demand curve, not by other firms' decisions.

[2] Jeanneret (2007) also points out the need to look at both timing and size of investment (in a somewhat different context, since he discusses FDI in the context of exporting goods versus exporting capital), stating: " . . . in order to gauge the overall effect of uncertainty on investment, it is more relevant to estimate the expected value of investment within a given time period." (p. 4).

and chosen optimally by the firm, similar to Bar-Ilan and Strange (1999) and Hagspiel et al. (2016).

1. Model Specifications

A firm holds an option to invest in a project. When it makes the investment, the project is implemented instantaneously, and production starts immediately. The size of the investment is given by the production capacity Q, and the cost of investment is φQ^{α} (as in Bar-Ilan and Strange, 1999, Bertola and Caballero, 1994, and Sarkar, 2009). We can think of φ as the unit cost of capital, where the amount of capital is given by Q^{α}; then α is a measure of the returns-to-scale of the technology, with α < (>) 1 representing increasing (decreasing) returns to scale. Since a higher α represents more decreasing returns to scale, we call α the decreasing-returns-to-s the decreasing-returns-to-scale parameter. Production cost is w per unit, and the output price is given by the inverse demand function:

$$p_t = y_t - \gamma q_t, \qquad (1)$$

where p_t is output price per unit, q_t is the output level, γ is a non-negative constant, and y_t is an exogenous shock to demand. This linear demand function (with uncertainty) is widely used in the literature, e.g., Hagspiel et al. (2016), Lederer and Mehta (2005), and Sarkar (2009). In equation (1), the parameter γ denotes the price-sensitivity of the product, and is a measure of the firm's market power. If $\gamma = 0$, demand is infinitely elastic, and the company has no market power; if γ is large, demand is highly inelastic, and the company has substantial market power.

The demand shock y can be viewed as the (random) strength of demand for the product. It is the state variable (source of uncertainty) in our model, as in Dangl (1999), Hagspiel et al. (2016), and Sarkar (2009), and evolves as a lognormal process:

$$dy/y = \mu dt + \sigma dZ, \qquad (2)$$

where μ and σ are the mean and standard deviation of the process, respectively, and Z is a standard Wiener process. This is a commonly used stochastic process in the literature (Dangl, 1999, Hagspiel et al., 2016, Sarkar, 2009). Thus, the uncertainty in our model is caused by demand volatility, which is common in real-option models (Tsai and Hung, 2009).

All free cash flows are paid to shareholders as dividends, and cash flows are discounted at a constant rate of r (we assume $r > \mu$, to ensure non-negative project values and to rule out speculative bubbles). The firm always operates at full capacity, i.e., $q_t = Q$, as in Bar-Ilan and Strange (1999), Lederer and Mehta (2005), Seta et al. (2012), etc. However, if the market turns too negative (that is, y falls sufficiently), the firm in our model does have the option to abandon operations and exit the business; this ensures that the model is consistent with limited liability.

2. Project Valuation

The instantaneous profit stream is given by revenues (pq) less operating costs (wq); thus, $\pi(q) = (y-w)Q - \gamma Q^2$. Then it can be shown that the project value is given by:[5]

$$V(y) = \frac{Qy}{(r-\mu)} - \frac{(w+\gamma Q)Q}{r} + Ay^{\theta_2} \qquad (3)$$

where A is a constant to be determined by boundary conditions, and θ_1 and θ_2 are solutions to the quadratic equation: 0r)1(5.02=−μθ+−θθσ, and are given by:

$$\theta_1 = 0.5 - \frac{\mu}{\sigma^2} + \sqrt{\left(0.5 - \mu/\sigma^2\right)^2 + \frac{2r}{\sigma^2}} \qquad \theta_2 = 0.5 - \frac{\mu}{\sigma^2} - \sqrt{\left(0.5 - \mu/\sigma^2\right)^2 + \frac{2r}{\sigma^2}}$$

[1] Note that the firm need not be a monopolist; all that is required for this analysis is that the firm have some market power or there be some product differentiation; in other words, all that is necessary is that the demand function be downward-sloping, e.g., as in monopolistic competition.

[2] In real life, firm can often produce at reduced rates (leaving some of the capacity idle) when demand is low. However, as Seta et al. (2012) explain, in many industries firms make production plans before the actual realization of market demand, and may find it difficult to produce below capacity due to fixed costs. Also, even when firms can keep some capacity idle, a temporary suspension of production is often costly, because of maintenance costs needed to avoid deterioration of the equipment.

[3] All derivations are standard in the real-options literature, hence they are not included in the paper. However, they are available on request from the author.

The first two terms of the project value in equation (3) represent the value of the profit stream from the project. The term Ay^{θ_2} represents the value of the option to abandon the project (therefore, $A \geq 0$).

3. Boundary Conditions and the Optimal Abandonment Trigger y_b

As mentioned above, when market conditions deteriorate sufficiently (say, when y falls to y_b), the firm will abandon the project; thus, y_b is the abandonment trigger. This provides two boundary conditions at the trigger $y = y_b$:

Value-matching (continuity) condition: $V(y_b) = 0$, and

Smooth-pasting (optimality) condition: $V'(y_b) = 0$.

The two boundary conditions can be solved for the constant A and the optimal abandonment trigger y_b:

$$A = -\frac{Q(y_b)^{1-\theta_2}}{\theta_2(r-\mu)} \quad \text{and} \quad y_b = \frac{(w+\gamma Q)(1-\mu/r)}{(1-1/\theta_2)}$$

4. The Investment Timing Decision

Consistent with the real-option literature, investing in the project is tantamount to exercising the option to invest. As is common in this literature, the option to invest is assumed to be a perpetual one, hence the value of the option will be a function of only the state variable y, say f(y). The firm will exercise this option (i.e., will invest) when y rises to a high enough level, say to $y = y_i$; then, y_i is the investment trigger.

Then it can be shown that the value of the option to invest is given by: $f(y) = Fy^{\theta_1}$, where F is a constant to be determined by boundary conditions. The optimal investment trigger y_i and the constant F can be determined from the boundary conditions at the trigger $y = y_i$:

Value-matching (continuity) condition: $f(y_i) = V(y_i) - \varphi Q^{\alpha}$

Smooth-pasting (optimality) condition: $f'(y_i) = V'(y_i)$

The boundary conditions give us a non-linear equation for the optimal investment trigger y_i:

$$\frac{Qy_i(1-1/\theta_1)}{(r-\mu)} - \frac{(w+\gamma Q)Q}{r} + A(1-\theta_2/\theta_1)(y_i)^{\theta_2} = \varphi Q^{\alpha} \tag{4}$$

Equation (4) has to be solved numerically for y_i, since it has no analytical solution. It is clear from equation (4) that the optimal investment trigger is a function of the size or capacity of the investment (Q).

5. The Capacity Decision

If the investment size or capacity Q is determined by the firm, it will choose Q so as to maximize the payoff at investment, which is: $\{V(y_i) - \varphi Q^{\alpha}\}$ or $\left\{\frac{Qy_i}{(r-\mu)} - \frac{(w+\gamma Q)Q}{r} + Ay_i^{\theta_2} - \varphi Q^{\alpha}\right\}$, where y_i is the state variable when the investment is made. Differentiating this expression with respect to Q and equating the derivative to zero gives us an equation that must be solved for the optimal capacity:

$$\frac{y_i}{(1-\mu/r)} - (w+\gamma Q) + \left[Q\gamma + \frac{w+\gamma Q}{1-\theta_2}\right]\left(\frac{y_i}{y_b}\right)^{\theta_2} = r\varphi\alpha Q^{\alpha-1} \tag{5}$$

Equation (5) is also solved numerically, since no analytical solution is available. From equation (5), it is clear that the optimal capacity is a function of the investment trigger y_i; in other words, optimal capacity is a function of when the investment is made.

6. The Expected Present Value of Investment (EPVI)

The fact that delayed (earlier) investment is larger (smaller) makes it difficult to identify the effect on investment with a one-dimensional measure, as pointed out in Section 1. We therefore use a composite measure that takes into

[6] Note that $\theta_1 > 1$ and $\theta_2 < 0$.

account both dimensions – timing and size. A logical way to do this would be to look at the EPV (expected present value) of the investment (in dollars) or EPVI, as mentioned in Section 1, since this incorporates both timing (by considering the present value) and size (by considering the size of investment in dollars). The EPVI is determined by the investment size, the investment trigger, and the parameters of the stochastic process governing y. The dollar amount to be invested when y rises to y_i is given by $\$\varphi Q^\alpha$. The expected (or probability-weighted) present value of φQ^α invested at the first passage time of y to y_i is given by ()1iyyQθαφ (see, for instance, Leland, 1994, footnote 16).

This is a direct measure of the effective magnitude of investment, in dollar terms, and takes into account uncertainty as well as the investment choice (both timing and size). Since it is in expected present value terms, it can be compared across various volatility scenarios; thus it should be an appropriate measure for our purpose. However, this measure varies continuously (since y is a continuous state variable); therefore, for more a meaningful comparison across different scenarios, we decided to get rid of y and just use the measure ()1iyQθαφ instead. Therefore, in the rest of the paper, we use this expression for the appropriate measure of investment:

$$\text{EPVI} = \frac{\varphi Q^{\alpha}}{(y_i)^{\theta_1}} \qquad (6)$$

Results

1. Base-case Parameter Values

Since the results are derived numerically, the values of the input parameters need to be specified. We start with a reasonable "base case" set of parameter values, and repeat the numerical procedure with a wide range of values, to ensure robustness of the results and to identify the comparative static relationships. The results are found to be valid for the entire range of parameter values examined. Thus, while it is not possible to guarantee that the results are completely general (because solutions are obtained numerically), we are confident that the results hold for reasonable economic scenarios.

The base-case parameter values are as follows: discount rate r = 5%, demand growth rate μ = 0%, and demand volatility σ = 15% (Sarkar, 2018). The price-sensitivity parameter γ = 1, unit cost of capital φ = 1, variable cost w = 1, and returns-to-scale parameter α = 1.

2. Optimal Investment Trigger for a Given Capacity, $y_i(Q)$

We first look at the standard real-option scenario, and compute the optimal investment trigger y_i for a given capacity Q, by solving equation (4) for different values of Q. Figure 1(a) illustrates the results with three different volatility levels, σ = 10%, 15% and 20%. There are two points worth noting in Figure 1(a): (i) y_i is an increasing function of Q (as expected), and (ii) y_i is also an increasing function of σ (consistent with the existing literature). Thus the "timing" effect of uncertainty on investment is negative. While these results are not new, they are included here for completeness and to highlight these two relationships.

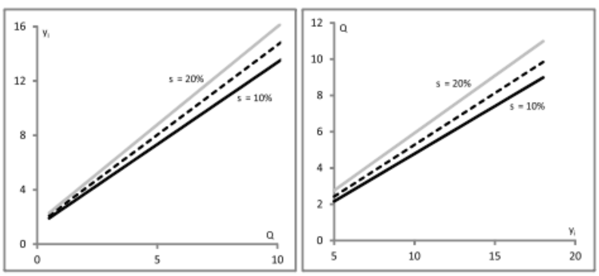


Figure 1. Part (a) shows optimal investment trigger y_i as a function of capacity Q, and part (b) shows Q as a function of y_i. In both cases, the relationship is shown for three volatility levels, σ = 10%, 15% and 20%, the broken line in between representing σ = 15%. The base-case parameter values are used: r = 5%, μ = 0%, γ = 1, φ = 1, w = 1 and α = 1.

[7] While this seems like an appropriate overall measure of investment, it is not yet in common use in the literature, and has been used only twice that we know of, by Nishihara et al. (2018) and Lukas and Thiergart (2016), although in a different context.

3. Optimal Capacity for Given Investment Trigger, $Q(y_i)$

We next look at optimal investment size Q as a function of the investment trigger y_i, by solving equation (5) for different values of y_i. The results are shown in Figure 1(b) for three different volatility levels, σ = 10%, 15% and 20%. Once again, there are two points worth noting: (i) Q is an increasing function of y_i, and (ii) Q is also an increasing function of σ. Thus the "size" effect of uncertainty on investment is positive.

The first finding is consistent with the finding of Bar-Ilan and Strange (1999), Hagspiel et al. (20216), Huberts et al. (2015), etc., that early (late) investment results in smaller (larger) investment. The second finding arises from the asymmetry inherent in limited liability (the option to abandon operations) and how it is affected by uncertainty. In our model, uncertainty is given by the volatility σ, and an increase in volatility means the state variable y is more likely to reach extreme values, both high and low. The firm will benefit from the high values of y (in the form of higher profits and value) but the downside from the low values is limited because of the abandonment option – if things turn too unfavorable the company will just exit the business. Thus, the negative consequences of a higher volatility are capped but the positive effects are not. This asymmetry means that, for higher volatility, the company is better off with larger capacity, to better take advantage of the higher highs, since limited liability largely insulates it from the disadvantages of the lower lows. Therefore, the optimal capacity is an increasing function of volatility or uncertainty.

4. Both Investment Trigger and Capacity Chosen Optimally

We next look at the case that is of most interest to us – what is the overall effect of uncertainty on investment (EPVI) when both investment timing and size are chosen optimally by the firm? The optimal investment trigger and optimal capacity are determined by solving equations (4) and (5) simultaneously, and the resulting EPVI computed from equation (6). With the base-case parameter values, we computed the results with three levels of volatility, as follows:

For σ = 10%, we get y_i = 2.0620, Q = 0.6256, and EPVI = 0.0430;
For σ = 15%, y_i = 3.5551, Q = 1.6030, and EPVI = 0.0544;
For σ = 20%, y_i = 11.3441, Q = 6.7771, and EPVI = 0.0359.

Figure 2 shows the results over a range of volatility. We note that both y_i and Q monotonically increasing functions of volatility. The former is a standard implication from option theory – that higher volatility delays investment. The latter relationship arises from the fact that higher volatility increases optimal capacity, as discussed in Section 3.3, as well as the fact that delayed investment is larger in size. Thus, the overall effect of a higher volatility is larger but delayed investment, since both y_i and Q are increasing in σ. While this is similar to Figure 1, both now rise at an increasing rate and the values are very high for large volatility. This happens because the two reinforce one another; a higher Q results in a higher y_i, which in turn results in a higher Q, and so on; hence the final y_i and Q are larger than in the earlier case. As a result, when volatility is very large, investment is usually postponed indefinitely (see Dangl, 1999). Also, EPVI falls to extremely low levels for large volatility. It is clear that high levels of uncertainty can be very unfavorable for corporate investment; not surprisingly, politicians are very reluctant to make dramatic policy changes once they achieve power, since such policy changes represent high business uncertainty.

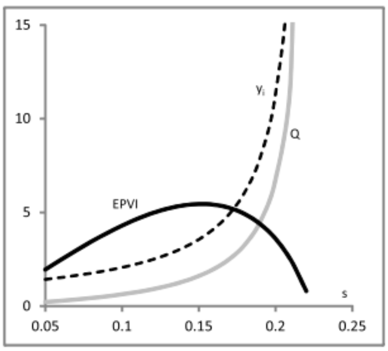


Figure 2. Shows optimal investment decision (y_i and Q) and the resulting EPVI as functions of volatility σ (note that the graph shows 100*EPVI). The base-case parameter values are used: r = 5%, μ = 0%, γ = 1, φ = 1, w = 1 and α = 1.

However, if the overall effect on investment is to be represented by a single (composite) measure, we look to the EPVI as a measure of investment. This is found to be a non-monotonic (inverted-U shaped) function of volatility, with EPVI initially increasing and subsequently decreasing in volatility. The non-monotonicity arises from the two conflicting effects of a higher volatility: (i) delayed investment (timing effect), which has a negative effect on investment because of discounting or time value, and (ii) larger capacity (size effect), which has a positive effect on investment. The discounting effect will be smaller and thus less important if investment is made early (i.e., y_i is low). As we can see from Figure 2, y_i is small when volatility is low. For small volatility, therefore, the discounting effect is small, and as a result the size effect dominates; therefore, EPVI is increasing in volatility when volatility is small. For large volatility, on the other hand, the investment trigger is higher, hence investment is delayed; this increases the importance of the timing effect, hence the timing effect dominates the size effect, and EPVI is decreasing in volatility. This explains why EPVI is an increasing function when volatility is low and a decreasing function when volatility is high.

Although the uncertainty-investment relationship is illustrated above only for the base case, the same relationship is observed in all cases (with all parameter combinations) examined, hence this is quite a general relationship. Extremes (too little or too much) in uncertainty do not help corporate investment, and the effect is particularly negative for very large volatility. The effect of uncertainty on investment is stated formally in Result 1 below.

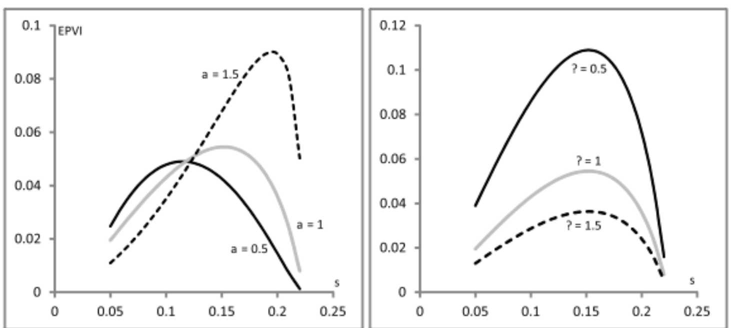
Result 1. When both the timing and size of investment are chosen optimally by the firm, the uncertainty-investment relationship is generally non-monotonic; investment (measured by EPVI) is initially increasing and subsequently decreasing in volatility.

Result 1 is the most important conclusion of our model. It is a new result because our measure of investment takes into account both timing and size of investment, hence it has not been empirically tested yet. Empirical testing would have to be done with a new way to estimate investment, since existing empirical studies have used measures that are not appropriate for this purpose, e.g., rate of capital expenditure per year. We leave the empirical testing of our result for future research.

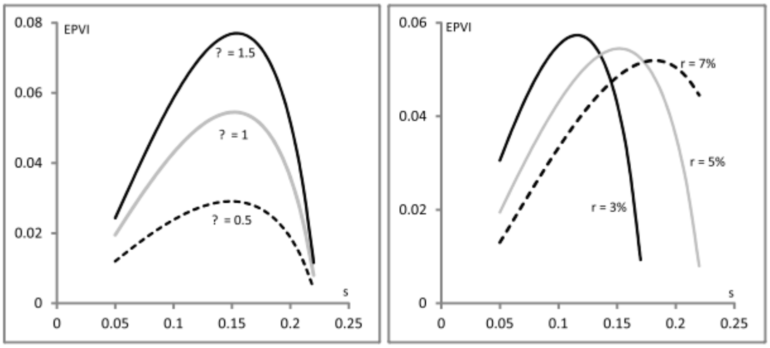
Although Result 1 provides the general shape of the uncertainty-investment relationship, it says nothing about the relative importance of the rising and falling sections. This might be important because it would affect the frequency with which we observe positive and negative relationships between uncertainty and investment. If, for instance, the point at which it switched from rising to falling was far to the right (i.e., high σ), we should more frequently observe an increasing relationship. Similarly, if it starts falling at low σ, we would more likely observe a negative relationship. In the next section, therefore, we repeat the numerical computations with different parameter values, to see (i) if the general shape is unchanged, and (ii) how the point at which the relationship turns from positive to negative varies with the different parameter values.

5. Comparative Static Results

Figure 3 displays the results with different parameter values. We note that *in all cases* examined, the general shape is the same (that is, an inverted-U shape); thus the main point of Result 1 (that investment is generally initially increasing and subsequently decreasing in uncertainty) is quite robust. However, there is significant variation in the exact point at which the relationship goes from positive to negative, as described in detail below.

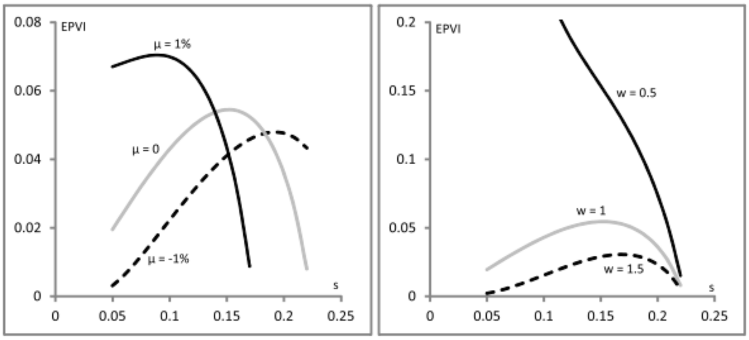


(a) (b)



(c) (d)

Figure 3. Comparative static results. Shows the effect of volatility σ on investment (EPVI) for a range of parameter values. Base-case parameter values: r = 5%, μ = 0%, γ = 1, φ = 1, w = 1 and α = 1.



(e) (f)

Figure 3 (continued)**.** Comparative static results.

<u>(a) Decreasing-returns-to-scale parameter α:</u> Figure 3(a) shows that the point at which EPVI starts falling varies greatly with α; for higher α EPVI starts falling later (at higher σ), and for lower α it starts falling earlier. Thus, the uncertainty-investment relationship is positive over a wider range of volatilities when α is higher; alternatively, we are more likely to observe an increasing relationship when α is higher. The explanation for this result lies in how the returns-to-scale parameter α impacts investment timing or y_i: a larger α increases the cost of investing and therefore results in a smaller Q, which results in a lower investment trigger y_i (or earlier investment); earlier investment reduces the impact of the discounting effect described in Section 3.4 and thereby causes EPVI to start falling later. Therefore, a higher (lower) α causes the EPVI curve to be increasing in σ over a wider (narrower) range of σ.

We can also note from Figure 3(a) that the effect of α on EPVI depends on the level of volatility; for small (large) volatility, EPVI is decreasing (increasing) in α. This is because a higher α (more decreasing returns to scale) results in a smaller Q (i.e., the size effect has a negative effect on investment) along with a lower y_i (i.e., the timing effect has a positive effect on investment). For small volatility, as shown above, the timing effect is smaller, hence the former effect above dominates, and the net effect of higher α on investment is negative. For large volatility, the discounting effect is stronger, hence the latter effect dominates, and the net effect of higher α on investment is positive. Thus, the overall effect of α on investment is ambiguous. This is very different from the traditional fixed-capacity real-option models, where a higher α has an unambiguous negative effect on investment. Finally, we also note that investment (EPVI) is more sensitive to volatility when α is high.

<u>(b) Price-sensitivity γ:</u> The point at which EPVI starts falling does not change with γ, hence γ has no effect on the shape of the uncertainty-investment relationship. Also, EPVI is a decreasing function of γ, which is not surprising since a higher γ

(greater sensitivity) means that greater output will be penalized more by drop in price (from equation (1)), resulting in a negative impact on investment. Finally, EPVI is more sensitive to volatility when γ is low.

(c) Unit investment cost φ: The point at which EPVI starts falling does not change with φ, hence unit investment cost has no impact on the shape of the uncertainty-investment relationship. As expected, EPVI is an increasing function of φ, since the dollar investment amount is directly and positively related to φ. Also, EPVI is more sensitive to volatility when φ is large.

(d) Discount rate r: The point at which EPVI starts falling varies with the discount rate: for higher r, EPVI starts falling later (at higher σ). Thus, the uncertainty-investment relationship is more likely to be positive when r is higher. The explanation is similar to that for α: with a higher discount rate r, the benefits from the investment (which come in the future) will be smaller but the investment cost (incurred at the beginning) will remain unchanged; thus, a higher r will result in a smaller optimal capacity. The smaller capacity results in earlier investment, which reduces the impact of the discounting effect, hence the EPVI starts falling at a later stage (for higher σ). Therefore, we are more likely to observe an increasing relationship when r is high.

We also note from Figure 3(d) that the effect of r on EPVI depends on the level of volatility; for small σ EPVI is decreasing in r, and for large σ EPVI is increasing in r. The explanation is similar to that for α: a higher r results in a smaller Q (i.e., the size effect has a negative effect on investment) and therefore also a lower y_i (i.e., the timing effect has a positive effect on investment). For small volatility, the timing effect is smaller, hence the former effect above dominates, and the net effect of higher r on investment is negative. For large volatility, the timing effect is stronger, hence the latter effect dominates, and the net effect of higher r is positive. Thus, the overall effect of r on investment is ambiguous, and depends on the volatility.

(e) Demand growth rate μ: The point at which EPVI starts falling varies with μ; for higher μ EPVI starts falling earlier (at lower σ). Thus the observed uncertainty-investment relationship is more likely to be negative (positive) when μ is higher (lower). The explanation is similar to that for α: a higher growth rate leads to larger capacity Q, and the larger capacity results in delayed investment, which increases the impact of the discounting effect. Hence, when μ is higher, EPVI starts falling earlier (for lower σ); that is, the relationship is more likely to be a negative one. Therefore, a higher (lower) μ raises the likelihood of observing a negative (positive) uncertainty-investment relationship.

The effect of μ on EPVI depends on the level of volatility; for small σ EPVI is increasing in μ, and for large σ EPVI is decreasing in μ. The explanation is the same as that for r above: a higher μ results in a larger Q (i.e., the size effect has a positive effect on investment) and therefore also a higher y_i (i.e., the timing effect has a negative effect on investment). For small volatility, the timing effect is smaller, hence the net effect of higher μ on investment is positive. For large volatility, the timing effect is stronger, hence the net effect of higher r is negative. Thus, the overall effect of r on investment is ambiguous, and depends on the volatility.

(f) Operating cost w: The point at which EPVI starts falling depends on w; for higher w, EPVI starts falling later (at higher σ). That is, the uncertainty-investment relationship is more likely to be positive (or is positive over a wider range of σ) when w is higher. The explanation is similar to the above: a higher production cost leads to a smaller capacity Q, and the smaller capacity results in earlier investment, which reduces the impact of the discounting effect. As a result, the EPVI starts falling later (when σ is higher). Therefore, a higher (lower) w causes the EPVI curve to be increasing in σ over a wider (narrower) range of σ. We also note that EPVI is a decreasing function of w, as expected. Finally, EPVI is more sensitive to volatility when w is small.

In Figure 3, it is also worth noting that investment is more sensitive to uncertainty in some situations, for instance, when the decreasing-returns-to-scale parameter and unit investment cost are high, or when operating cost and price sensitivity are low. The comparative static results are summarized below.

Result 2.

(a) While the effect of uncertainty on investment might be positive or negative, it is more likely to be positive for low demand growth rate and volatility, and high decreasing-returns-to-scale parameter, discount rate and operating cost.

(b) Investment is more sensitive to volatility when the decreasing-returns-to-scale parameter and unit investment cost are high, and when operating cost and price sensitivity are low.

One reason there is so much interest in the effect of uncertainty is that uncertainty is substantially impacted by government policies. If government policies are unstable (changed frequently), economic and market uncertainty increases. What the above results suggest is that for high-margin (low w), high growth, high-capital-cost (high φ), competitive (low γ) and decreasing-returns-to-scale (high α) projects, government policies are more relevant because investment is more sensitive to uncertainty. In these cases, therefore, government decisions and policies have larger impact on corporate investment, and have to be made more carefully.

Summary and Conclusions

This paper uses a real-option model to study how uncertainty affects corporate investment. Existing real-option models focus on the investment timing choice, assuming a fixed-size investment; but in real life, the firm generally gets to choose investment size as well. We therefore examine the case when both the timing and the size of the investment are chosen by the firm. In such a scenario, however, the effect on investment has to capture both size and timing, hence a new composite measure of investment is necessary. We propose such a measure, the expected present value of investment (in dollars) or EPVI.

We show that investment is, in general, a non-monotonic function, initially increasing and subsequently decreasing, in uncertainty. Thus, uncertainty can have a positive or negative effect on investment; however, the point at which it switches from positive to negative can vary widely with input parameter values such as discount rate or demand growth. Therefore, for some ranges of parameter values, we are more likely to observe a positive relationship between uncertainty and investment, such as low demand growth rate and volatility, or high discount rate, operating cost and decreasing-returns-to-scale. In these situations, therefore, greater uncertainty is good for investment. We also show that how certain parameters impact investment can differ greatly from the fixed-capacity case, and may depend on the level of uncertainty; for instance, investment is decreasing (increasing) in the discount rate when the level of uncertainty is low (high).

As in most models, we make simplifying assumptions for reasons of tractability. For instance, we consider only one investment opportunity that requires a lumpy investment, with no option of future expansion. Thus the model will be useful for lumpy, one-shot investments, but not for investments with expansion opportunities or for a company considering multiple investment opportunities at the same time. However, this is common to many real-option models.

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